

Quadratics Study Guide

Key

To be successful on your test should know...

a) How to find the Axis of Symmetry

$$AOS = \frac{-b}{2a}$$

Example: Find the axis of symmetry for the equation $y = 3x^2 + 9x - 5$

$$\frac{-9}{2(3)} = \frac{-9}{6} = -1.5 \quad \boxed{x = -1.5}$$


b) How to use the Axis of Symmetry to find the vertex


Since the vertex is on the AOS, plug your AOS into your equation for x and solve for y .

Example: Using the axis of symmetry from part A, what is the vertex of the equation?

$$y = 3(-1.5)^2 + 9(-1.5) - 5$$
$$y = 6.75 - 13.5 - 5 = -11.75 \quad \boxed{\text{vertex } (-1.5, -11.75)}$$

c) How to know if the vertex will be a maximum or a minimum

if a (the coefficient in front of x^2) is positive  minimum

if a is negative  maximum

Example: Is the vertex from part B a minimum or a maximum

Minimum

d) How to find the domain and range based on the vertex

domain is always all real numbers

range if minimum is $y \geq$ the y value of the minimum

range if maximum is $y \leq$ the y value of the maximum

Example: What is the domain and range for the equation in part A? Use the minimum or maximum that you found.

domain: all real numbers
range: $y \geq -11.75$

e) The formula used for throwing/dropping something that describes its height over time

$$h = -16t^2 + vt + c$$

Example: Mr. Chevy threw a football across the bus lot. He released the ball 5 feet above the ground. The football was thrown with a velocity of 20 ft/sec. Write an expression that describes the football's height over time.

$$h = -16t^2 + 20(t) + 5$$

f) How to find the maximum height for an object that is thrown/dropped

Find the vertex! (t, h)

Example: What is the maximum height of the football Mr. Chevy threw?

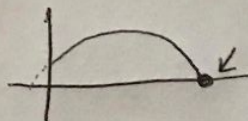
$$AOS = \frac{-20}{2(-16)} = 0.625 \quad (0.625, 11.25)$$

$$h = -16(0.625)^2 + 20(0.625) + 5 = -6.25 + 12.5 + 5 = 11.25$$

$$\text{max height} = 11.25 \text{ ft}$$

g) How to find when the object that is thrown/dropped will hit the ground

it is the larger x-intercept



Example: Assuming that Mr. Chevy's ball doesn't hit anything on its journey, when will it hit the ground?

$$\frac{-20 \pm \sqrt{(20)^2 - 4(-16)(5)}}{2(-16)} = \frac{-20 \pm \sqrt{720}}{-32} = \frac{-20 \pm 26.83}{-32} = -0.21 \text{ or } \underline{1.45}$$

$$\underline{1.45 \text{ seconds}}$$

h) How to solve quadratics by

i) Graphing

graph and find the x-intercepts

$$\begin{array}{r|l} x & y \\ 2 & 0 \\ 1 & 0 \end{array}$$

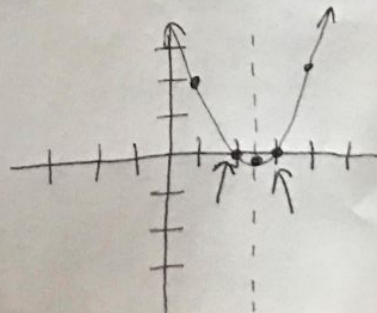
Example: Solve $m^2 - 5m + 4 = -2$ by graphing

$$m^2 - 5m + 4 = 0$$

$$AOS = \frac{5}{2} = 2.5$$

$$\text{vertex } (2.5, -2.25)$$

$$\begin{array}{l} x=2 \\ x=3 \end{array}$$



ii) Factoring

- ① Set equal to 0
- ② Factor
- ③ Use zero product property

Example: Solve $n^2 + 3n - 12 = 6$ by factoring

$$n^2 + 3n - 18 = 0$$

$$(n-6)(n+3) = 0$$

$$n-6=0 \quad n+3=0$$

$$\boxed{n=6} \quad \text{or} \quad \boxed{n=-3}$$

iii) Completing the square

- ① make sure a is 1
- ② get rid of anything in your way
- ③ use $(\frac{b}{2})^2$ to make a perfect square trinomial
- ④ solve for x

Example: Solve $x^2 - 10x + 26 = 8$ by completing the square

$$x^2 - 10x = -18$$

$$x^2 - 10x + 25 = -18 + 25$$

$$(x-5)^2 = 7$$

$$x-5 = \pm 2.65$$

$$x-5 = 2.65 \quad \text{or} \quad x-5 = -2.65$$

$$\boxed{x = 7.65} \quad \text{or} \quad \boxed{x = 2.35}$$

$$\left(\frac{-10}{2}\right)^2 = 25$$

iv) Using the quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: Solve $2k^2 + 9k = -7$ using the quadratic formula

$$2k^2 + 9k + 7 = 0$$

$$\frac{-9 \pm \sqrt{9^2 - 4(2)(7)}}{2(2)} = \frac{-9 \pm \sqrt{25}}{4} = \frac{9 \pm 5}{4}$$

$$\boxed{x=3.5} \quad \text{or} \quad \boxed{x=1}$$

i) What is the discriminant and what does it tell you

$b^2 - 4ac \rightarrow$ tells you the # of solutions

if positive \rightarrow 2 solutions

if negative \rightarrow 0 solutions

if zero \rightarrow 1 solution

Example: Find the discriminant of the quadratic $5x^2 + 9x = -4$. What does this tell you?

$$5x^2 + 9x + 4 = 0$$

$$9^2 - 4(5)(4) = 81 - 80 = \boxed{1} \rightarrow \boxed{2 \text{ solutions}}$$

j) What is standard form of a quadratic

$$ax^2 + bx + c = 0$$

Example: Write $y = -3(x-2)^2 - 4$ in standard form.

$$\begin{aligned}y &= -3(x-2)(x-2) - 4 \\y &= -3(x^2 - 4x + 4) - 4 \\y &= -3x^2 + 12x - 12 - 4\end{aligned}$$

$$y = -3x^2 + 12x - 16$$

k) What is vertex form of a quadratic

$$y = a(x-h)^2 + k$$

Example: Write $y = (x+5)(x+4)$ in ^{vertex} standard form

$$y = x^2 + 9x + 20$$

$$AOS = \frac{-9}{2} = -4.5$$

$$\text{vertex } (-4.5, .5)$$

$$y = 1(x + 4.5)^2 + .5$$

l) How do you find the vertex of a quadratic on your calculator

2nd \rightarrow trace \rightarrow maximum/minimum \rightarrow left bound, right bound, guess

Example: Find the vertex of $y = 3x + 4x^2 - 2$ using your graphing calculator.

$$(-.375, -2.563)$$

m) How do you find a solution of a quadratic on your calculator

2nd \rightarrow trace \rightarrow zero \rightarrow left bound, right bound, guess

Example: What are the solutions to the quadratic that you graphed in part L? Use your calculator to solve.

$$-1.175 \quad \text{and} \quad .425$$

n) How to solve a system of linear and quadratic equations by

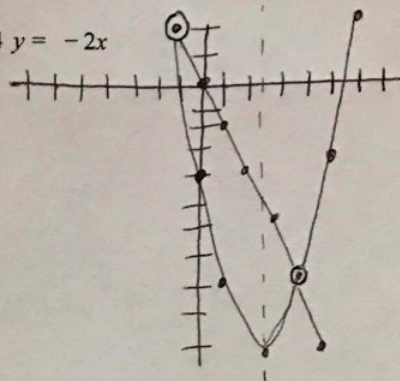
i) Graphing

Graph both equations and find the intersection

Example: Solve the system by graphing

$$y = x^2 - 5x - 4 \quad y = -2x$$

$$(4, -8) \text{ and } (-1, 2)$$



ii) Substitution

- ① substitute for y
- ② simplify quadratic
- ③ solve quadratic
- ④ plug back in

Example: Solve the system by substitution $-x^2 - x + 19 = y$ $x = y + 80$

$$\begin{aligned} x &= -x^2 - x + 19 + 80 \\ 0 &= -x^2 - 2x + 99 \\ 0 &= -(x^2 + 2x - 99) \\ 0 &= -(x + 11)(x - 9) \end{aligned}$$

$$\begin{array}{l|l} x = -11 & x = 9 \\ -11 = y + 80 & 9 = y + 80 \\ y = -91 & y = -71 \end{array}$$

$$\begin{array}{l} (-11, -91) \\ (9, -71) \end{array}$$

iii) Elimination

- ① eliminate the y
- ② solve the quadratic
- ③ plug back in

Example: Solve the system by elimination $y = 5x - 20$

$$\begin{array}{r} y = x^2 - 5x + 5 \\ -(y = 5x - 20) \\ \hline 0 = x^2 - 10x - 25 \end{array}$$

$$\begin{aligned} 0 &= (x - 5)^2 \\ 0 &= x - 5 \\ 5 &= x \end{aligned}$$

$$\begin{aligned} y &= x^2 - 5x + 5 \\ y &= 5(5) - 20 \\ y &= 25 - 20 \\ y &= 5 \end{aligned}$$

$$(5, 5)$$

iv) Graphing calculator

- ① graph both equations in Y=
- ② 2nd → trace → intersect → enter 3 times on the intersection

Example: Solve the system using your graphing calculator $y = -.5x^2 - 2x + 1$ $y + 3 = -x$

$$(-4, 1) \text{ and } (2, -5)$$