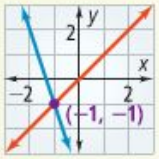
6 Chapter Review

Connecting BIG ideas and Answering the Essential Questions

1 Solving Equations and Inequalities

There are several ways to solve systems of equations and inequalities, including graphing and using equivalent forms of equations and inequalities within the system. The number of solutions depends on the type of system.

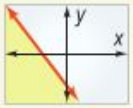
Solving Systems of Equations (Lessons 6-1, 6-2, and 6-3)



$y = x$
$y = -3x - 4$

The solution is $(-1, -1)$.

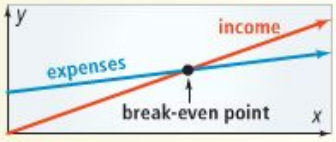
Linear Inequalities (Lessons 6-5 and 6-6)



2 Modeling

You can represent many real-world mathematical problems algebraically. When you need to find two unknowns, you may be able to write and solve a system of equations.

Applying Linear Systems (Lesson 6-4)



Chapter Vocabulary

- consistent (p. 361)
- dependent (p. 361)
- elimination method (p. 374)
- inconsistent (p. 361)
- independent (p. 361)
- linear inequality (p. 390)
- solution of an inequality (p. 390)
- solution of a system of linear equations (p. 360)
- solution of a system of linear inequalities (p. 396)
- substitution method (p. 368)
- system of linear equations (p. 360)
- system of linear inequalities (p. 396)

Choose the correct term to complete each sentence.

1. A system of equations that has no solution is said to be ?.
2. You can solve a system of equations by adding or subtracting the equations in such a way that one variable drops out. This is called the ? method.
3. Two or more linear equations together form a(n) ?.

6-1 Solving Systems by Graphing

Quick Review

One way to solve a system of linear equations is by graphing each equation and finding the intersection point of the graph, if one exists.

Example

What is the solution of the system? $y = -2x + 2$, $y = 0.5x - 3$

$y = -2x + 2$ Slope is -2; y-intercept is 2.

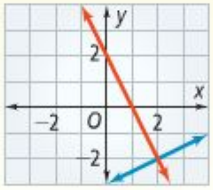
$y = 0.5x - 3$ Slope is 0.5; y-intercept is -3.

The lines appear to intersect at $(2, -2)$. Check if $(2, -2)$ makes both equations true.

$-2 = -2(2) + 2$ ✔

$-2 = 0.5(2) - 3$ ✔

So, the solution is $(2, -2)$.



Exercises

Solve each system by graphing. Check your answer.

4. $y = 3x + 13$, $y = x - 3$

5. $y = -x + 4$, $y = 3x + 12$

6. $y = 2x + 3$, $y = \frac{1}{3}x - 2$

7. $y = 1.5x + 2$, $4.5x - 3y = -9$

8. $y = -2x - 21$, $y = x - 7$

9. $y = x + 1$, $2x - 2y = -2$

10. Songwriting Jay has written 24 songs to date. He writes an average of 6 songs per year. Jenna started writing songs this year and expects to write about 12 songs per year. How many years from now will Jenna have written as many songs as Jay? Write and graph a system of equations to find your answer.

11. Reasoning Describe the graph of a system of equations that has no solution.

6-2 Solving Systems Using Substitution

Quick Review

You can solve a system of equations by solving one equation for one variable and then substituting the expression for that variable into the other equation.

Example

What is the solution of the system? $y = -\frac{1}{3}x$, $3x + 3y = -18$

$3x + 3y = -18$	Write the second equation.
$3x + 3(-\frac{1}{3}x) = -18$	Substitute $-\frac{1}{3}x$ for y.
$2x = -18$	Simplify.
$x = -9$	Solve for x.
$y = -\frac{1}{3}(-9)$	Substitute -9 for x in the first equation.
$y = 3$	

The solution is $(-9, 3)$.

Exercises

Solve each system using substitution. Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

12. $y = 2x - 1$, $2x + 2y = 22$

13. $-x + y = -13$, $3x - y = 19$

14. $2x + y = -12$, $-4x - 2y = 30$

15. $\frac{1}{3}y = \frac{7}{3}x + \frac{5}{3}$, $x - 3y = 5$

16. $y = x - 7$, $3x - 3y = 21$

17. $3x + y = -13$, $-2x + 5y = -54$

18. Business The owner of a hair salon charges \$20 more per haircut than the assistant. Yesterday the assistant gave 12 haircuts. The owner gave 6 haircuts. The total earnings from haircuts were \$750. How much does the owner charge for a haircut? Solve by writing and solving a system of equations.

6-3 and 6-4 Solving Systems Using Elimination; Applications of Systems

Quick Review

You can add or subtract equations in a system to eliminate a variable. Before you add or subtract, you may have to multiply one or both equations by a constant to make eliminating a variable possible.

Example

What is the solution of the system? $3x + 2y = 41$, $5x - 3y = 24$

$3x + 2y = 41$ — Multiply by 3. → $9x + 6y = 123$

$5x - 3y = 24$ — Multiply by 2. → $10x - 6y = 48$

$19x + 0 = 171$

$x = 9$

$3x + 2y = 41$ Write the first equation.

$3(9) + 2y = 41$ Substitute 9 for x.

$y = 7$ Solve for y.

The solution is (9, 7).

Exercises

Solve each system using elimination. Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

19. $x + 2y = 23$, $5x + 10y = 55$

20. $7x + y = 6$, $5x + 3y = 34$

21. $5x + 4y = -83$, $3x - 3y = -12$

22. $9x + \frac{1}{2}y = 51$, $7x + \frac{1}{3}y = 39$

23. $4x + y = 21$, $-2x + 6y = 9$

24. $y = 3x - 27$, $x - \frac{1}{3}y = 9$

25. Flower Arranging It takes a florist 3 h 15 min to make 3 small centerpieces and 3 large centerpieces. It takes 6 h 20 min to make 4 small centerpieces and 7 large centerpieces. How long does it take to make each small centerpiece and each large centerpiece? Write and solve a system of equations to find your answer.

Answers

Chapter Review

1. inconsistent **2.** elimination

3. system of linear equations

4. (−8, −11) **5.** (−2, 6)

6. (−3, −3) **7.** no solution

8. $\left(-\frac{14}{3}, -\frac{35}{3}\right)$

9. infinitely many solutions

10. 4 yr

11. The lines will be parallel.

12. (4, 7) **13.** (3, −10)

14. no solution **15.** (−1, −2)

16. infinitely many solutions

17. $\left(-\frac{11}{17}, -\frac{188}{17}\right)$ **18.** $55

19. no solution **20.** (−1, 13)

21. (−11, −7) **22.** (5, 12)

23. (4.5, 3)

24. infinitely many solutions

25. small centerpiece: 25 min, large centerpiece: 40 min

6 Chapter Test

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SKIP NUMBER 10, 11, 12, 13, 17,

Do you know HOW?

Solve each system by graphing. Tell whether the system has *one solution, infinitely many solutions,* or *no solution.*

1. $y = 3x - 7$
   $y = -x + 1$
2. $x + 3y = 12$
   $x = y - 8$
3. $x + y = 5$
   $x + y = -2$

Solve each system using substitution.

4. $y = 4x - 7$
   $y = 2x + 9$
5. $8x + 2y = -2$
   $y = -5x + 1$
6. $y + 2x = -1$
   $y - 3x = -16$

Solve each system using elimination.

7. $4x + y = 8$
   $-3x - y = 0$
8. $2x + 5y = 20$
   $3x - 10y = 37$
9. $3x + 2y = -10$
   $2x - 5y = 3$

Solve each system of inequalities by graphing.

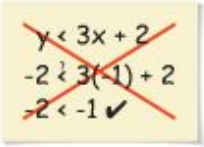
10. $y > 4x - 1$
    $y \leq -x + 4$
11. $x > -3$
    $-3x + y \geq 6$
12. **Garage Sale** You go to a garage sale. All the items cost \$1 or \$5. You spend less than \$45. Write and graph a linear inequality that models the situation.
13. **Gardening** A farmer plans to create a rectangular garden that he will enclose with chicken wire. The garden can be no more than 30 ft wide. The farmer would like to use at most 180 ft of chicken wire.
    a. Write a system of linear inequalities that models this situation.
    b. Graph the system to show all possible solutions.

Write a system of equations to model each situation. Solve by any method.

14. **Education** A writing workshop enrolls novelists and poets in a ratio of 5 : 3. There are 24 people at the workshop. How many novelists are there? How many poets are there?
15. **Chemistry** A chemist has one solution containing 30% insecticide and another solution containing 50% insecticide. How much of each solution should the chemist mix to get 200 L of a 42% insecticide?

Do you UNDERSTAND?

16. **Open-Ended** Write a system of two linear equations that has no solution.
17. **Error Analysis** A student concluded that $(-2, -1)$ is a solution of the inequality $y < 3x + 2$, as shown below. Describe and correct the student's error.



18. **Reasoning** Consider a system of two linear equations in two variables. If the graphs of the equations are not the same line, is it possible for the system to have infinitely many solutions? Explain.

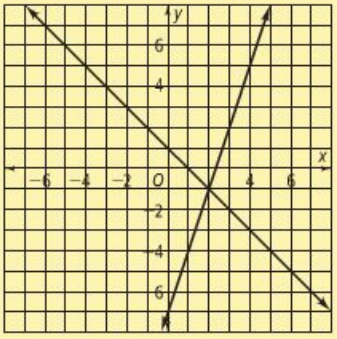
Reasoning Suppose you add two linear equations that form a system, and you get the result shown below. How many solutions does the system have?

19. $x = 8$
20. $0 = 4$
21. $0 = 0$

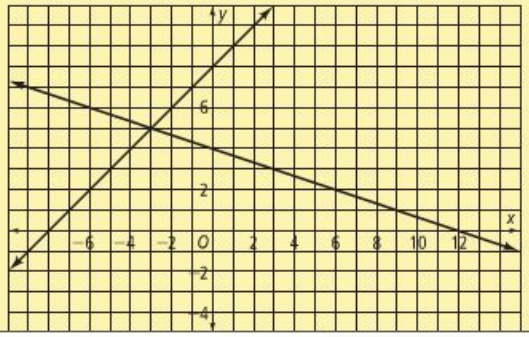
Answers

Chapter Test

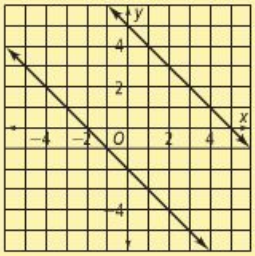
1. one solution: (2, −1)



2. one solution: (−3, 5)



3. no solution



4. (8, 25)

5. (2, −9)

6. (3, −7)

7. (8, −24)

8. (11, −0.4)

9. $\left(-\frac{44}{19}, -\frac{29}{19}\right)$

14. 15 novelists; 9 poets

15. 80 L of the 30% insecticide, 120 L of the 50% insecticide

16. Check students' work.

18. No; two lines can intersect in no points, one point, or infinitely many points. If they intersect in infinitely many points, then the two lines must be the same line.

19. one solution **20.** no solution

21. infinitely many solutions